

Hong Kong Mathematics Olympiad (2016/2017)
Final Event 1 (Group)

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
			<hr/>			
			Total score			<input type="text"/>

Team No.	<input type="text"/>
Time	<input type="text"/>
	<input type="text"/>
Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特別聲明，答案須用數字表達，並化至最簡。

1. If real numbers x , y and z satisfy $x + \frac{1}{y} = -1$, $y + \frac{1}{z} = -2$ and $z + \frac{1}{x} = -5$, determine the value of

$$a = \frac{1}{xyz}.$$

若實數 x 、 y 及 z 滿足方程 $x + \frac{1}{y} = -1$ ， $y + \frac{1}{z} = -2$ 及 $z + \frac{1}{x} = -5$ ，求 $a = \frac{1}{xyz}$ 的值。

$$a =$$

2. If $|x - |2x - 1|| = \frac{1}{2}$ is a real equation, determine the value of b , the number of real solutions of the equation.

若 $|x - |2x - 1|| = \frac{1}{2}$ 為實數方程，求實根數量 b 的值。

$$b =$$

3. If real numbers x and y satisfy $xy > 0$ and $x + y = 3$, find c , the maximum value of $\left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{y}\right)$.

若實數 x 及 y 滿足 $xy > 0$ 及 $x + y = 3$ ，求 $\left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{y}\right)$ 的最大值 c 。

$$c =$$

4. If a real number x satisfies $x - \frac{1}{x} = 3$, determine the value of $d = x^5 - \frac{1}{x^5}$.

若實數 x 滿足 $x - \frac{1}{x} = 3$ ，求 $d = x^5 - \frac{1}{x^5}$ 的值。

$$d =$$

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Final Event 2 (Group)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. In base-6 system, if $12345_6 \div 13_6$ has remainder A , determine the value of A .

在六進制中，若 A 為 $12345_6 \div 13_6$ 的餘數，求 A 值。

$A =$

2. Any two vertices in a cube can form a line segment. If B is the greatest number of line segments thus formed, determine the value of B .

立方體的任意兩個頂點可相連成一線段。若 B 為最多所能夠相連成的直線的數量，求 B 的值。

$B =$

3. If real numbers x , y and z satisfy $x + y + z = 30$ and $C = x^2 + y^2 + z^2$, determine the least value of C .

若實數 x , y 及 z 滿足 $x + y + z = 30$ 及 $C = x^2 + y^2 + z^2$ 。求 C 的最小值。

$C =$

4. If $y = (x-1)^3 + 3$, determine the greatest value of y for $-3 \leq x \leq 3$.

若 $y = (x-1)^3 + 3$ ，當 $-3 \leq x \leq 3$ ，求 y 的最大值。

$y =$

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Final Event 3 (Group)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Let a , b and c be integers with $1 < a < b < c$. If $(ab-1)(bc-1)(ac-1)$ is divisible by abc , determine the value of remainder R of $ab+bc+ac-1$ divided by abc .

設 a , b 及 c 為整數且 $1 < a < b < c$ 。若 $(ab-1)(bc-1)(ac-1)$ 可被 abc 整除，求 $ab+bc+ac-1$ 除以 abc 所得之餘數 R 的值。

$R =$

2. If $0 < x < 1$, determine the value of $S = \left(\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2}+x-1} \right) \cdot \left(\sqrt{\frac{1}{x^2}-1} - \frac{1}{x} \right)$.

若 $0 < x < 1$ ，求 $S = \left(\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2}+x-1} \right) \cdot \left(\sqrt{\frac{1}{x^2}-1} - \frac{1}{x} \right)$ 的值。

$S =$

3. Determine the value of T , the sum of real roots of $x^4 + (x-4)^4 = 544$.

求方程 $x^4 + (x-4)^4 = 544$ 的實根之和 T 的值。

$T =$

4. In triangle ABC , $BC = a$, $\angle ABC = \frac{\pi}{3}$ and its area is $\sqrt{3} \cdot a^2$. Determine the value of $U = \tan \angle ACB$.

在三角形 ABC 中， $BC = a$ ， $\angle ABC = \frac{\pi}{3}$ 及面積為 $\sqrt{3} \cdot a^2$ 。求 $U = \tan \angle ACB$ 的值。

$U =$

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Final Event 4 (Group)

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除非特別聲明，答案須用數字表達，並化至最簡。

1. To make a specific toy, it must be first moulded and then painted. Mr. A can mould 3 pieces of toys or paint 15 pieces of toys in one day, whereas Mr. B can mould 5 pieces or paint 15 pieces of toys in one day. Each of them can either mould or paint toys in one day, but not both. If Mr. A and Mr. B work together, determine the least number of days P to make 120 toys.

製作某玩具，需要先倒模，後上色。甲先生每日可以為 3 件玩具倒模，或為 15 件玩具上色；乙先生每日則可以為 5 件玩具倒模，或為 15 件玩具上色。若甲先生和乙先生合作，求最少多少日 P 才可以製作出 120 件玩具。

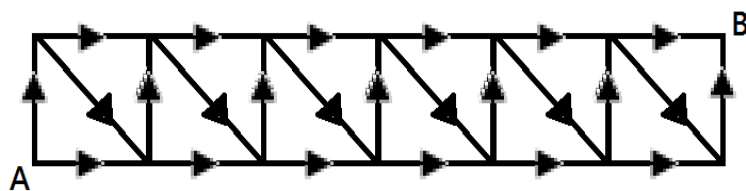
$$P =$$

2. In a duck shooting game, a boy fires 10 shots. The probability of him shooting down a duck with a shot is 0.5. Determine the probability Q of him shooting down the 6th duck at the last shot.

在一個射鴨遊戲中，一男孩射了 10 發子彈，該男孩每發子彈射中鴨子的概率為 0.5。求他於最後一發子彈射中第六隻鴨子的概率 Q 。

$$Q =$$

3. Determine the number of ways R getting from point A to B with the direction indicated by the arrows.

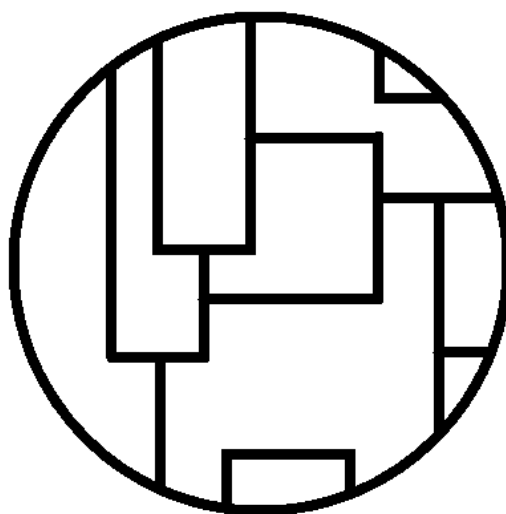


求按箭咀方向由 A 往 B 的路線總數 R 。

$$R =$$

4. To shade all the regions inside the following circular map using 3 colours, for which adjacent regions must not be in the same colour. Determine the maximum number S of regions being shaded by the same colour.

如果用 3 款顏料替下圖中所有區域著色，並且相鄰的區域不可用相同顏料。求同一款顏料最多可用作上色的區域數目 S 。



$S =$